The f_{D_s} Puzzle

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Abstract

Recent measurements of the branching fraction for $D_s \to \ell\nu$ disagree with the Standard Model by around 2σ. In this case the key aspect of the Standard Model is the calculation of the decay constant, f_{D_s}, with lattice QCD. This talk surveys the experimental measurements, and explains how the lattice QCD calculations are done. Should the discrepancy strengthen again (it was earlier 3.8σ), it would be a signal of new physics. Models that could explain such an effect are also discussed.

1. Introduction

The decay constant of a meson parametrizes the decay of the meson to leptons. For a pseudoscalar like the charmed strange meson D_s, it is defined by the hadronic matrix elements

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle = if_{D_s}p_\mu, \qquad (1)$$

$$(m_c + m_s)\langle 0|\bar{s}\gamma_5 c|D_s\rangle = -m_{D_s}^2 f_{D_s}. \qquad (2)$$

The Ward identity of the partially conserved axial current (PCAC) ensures that the two definitions are identical. These matrix elements are directly computable in QCD, via numerical simulations of lattice gauge theory. These calculations are useful: the ratio f_π/f_K, for example, is used to determine the Cabibbo angle, $\tan\theta_C \propto [B(K \to l\nu)/B(\pi \to l\nu)]^{1/2} f_\pi/f_K$ [1].

In the Standard Model, the expression for the branching ratio is

$$B(D_s \to \ell\nu) = \frac{m_{D_s}\tau_{D_s}}{8\pi} f_{D_s}^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 |G_F V_{cs}^* m_\ell|^2, \qquad (3)$$

with analogous expressions for leptonic decays of other pseudoscalar mesons. Here G_F is the Fermi constant, measured in muon decay, and V is the CKM matrix. Beyond the Standard Model, one replaces

$$G_F V_{cs}^* m_\ell \mapsto G_F V_{cs}^* m_\ell + G_A^\ell m_\ell + G_P^\ell \frac{m_{D_s}^2}{m_c + m_s}, \qquad (4)$$

where G_A^ℓ and G_P^ℓ are related to couplings and masses of new interactions in a way analogous to $G_F = g^2/\sqrt{2}M_W^2$. Amplitudes that proceed through an axial-vector current (G_F and G_A) are helicity suppressed, but amplitudes that proceed through a pseudoscalar current (G_P) are not. In practice, $D_s \to \mu\nu$ has a helicity-suppression factor $m_\mu^2/m_{D_s}^2 = 2.8 \times 10^{-3}$. On the other hand, $D_s \to \tau\nu$ is not helicity suppressed but, instead, phase-space suppressed: $(1 - m_\tau^2/m_{D_s}^2)^2 = 3.4 \times 10^{-2}$.

The decay constant of the D_s was expected to be an excellent test of lattice QCD, for several reasons [2]. The matrix elements in Eqs. (1) and (2) are gold-plated, in the sense of Ref. [3], namely, only one hadron enters, and the chiral extrapolation is controlled. Experimental measurements of $|V_{cs}|f_{D_s}$, via Eq. (3), can be combined with the determination of $|V_{cs}|$ from CKM unitarity. The idea that new physics could compete is usually discounted, because the decay is Cabibbo-favored and

proceeds at the tree level of the weak interactions. Finally, the precision of experiments has lagged that of calculations, so the analysis of the numerical lattice-QCD data is carried out with a relatively blind eye.

The first round of testing seemed to go well. In June 2005, the first lattice-QCD calculation with 2+1 flavors of sea quarks appeared in a joint work of the Fermilab Lattice and MILC Collaborations:

$$f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV Fermilab/MILC [4]}, \qquad (5)$$

where the first error is statistical and the second systematic. This prediction was followed a year later by a comparably accurate measurement of $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$ that, when combined with an independent measurement of $B(D_s \to \phi\pi)$, yielded

$$f_{D_s} = 283 \pm 17 \pm 7 \pm 14 \text{ MeV } \mu\nu/\phi\pi \text{ BaBar [5]}, \qquad (6)$$

which agrees with Eq. (5) at 1.2σ. Both results are discussed in more detail below.

But then something unexpected happened. During 2007 the CLEO and Belle Collaborations both published absolute measurements of $B(D_s \to \mu\nu)$ [6, 7], and CLEO also published absolute measurements of $B(D_s \to \tau\nu)$ [6, 8]. Transcribed via Eq. (3) as measurements of the decay constant, these are

$$f_{D_s} = 264 \pm 15 \pm 7 \text{ MeV} \quad \mu\nu \quad \text{CLEO [6]}, \qquad (7)$$

$$f_{D_s} = 275 \pm 16 \pm 12 \text{ MeV} \quad \mu\nu \quad \text{Belle [7]}, \qquad (8)$$

$$f_{D_s} = 310 \pm 25 \pm 8 \text{ MeV} \quad \tau\nu \quad \text{CLEO [6]}, \qquad (9)$$

$$f_{D_s} = 273 \pm 16 \pm 8 \text{ MeV} \quad \tau\nu \quad \text{CLEO [8]}. \qquad (10)$$

Taking Eqs. (6)–(10) at face value, the weighted average (combining all errors in quadrature) is [9]

$$f_{D_s} = 277 \pm 9 \text{ MeV}, \qquad (11)$$

which is 1.5σ higher than the value in Eq. (5). Meanwhile, and more dramatically, the HPQCD Collaboration published a lattice-QCD calculation with an error significantly smaller than Fermilab/MILC's:

$$f_{D_s} = 241 \pm 3 \text{ MeV HPQCD [10]}. \qquad (12)$$

Section 3. explains why the error is so much smaller. The difference between Eqs. (11) and (12) is 3.8σ. (Omitting BaBar, as in Ref. [11], the discrepancy becomes 3.4σ.) It is important to bear in mind—and easy to see from Eqs. (11) and (12)—that the yardstick for σ is the experimental statistical error. The 2008 edition of the Review of Particle Physics [11] noted that the discrepancy could be a sign of physics beyond the Standard Model. Candidate models are discussed below.

The rest of this paper brings this saga up to date. Section 2. gives a brief survey of the experiments, including higher-statistics measurements from CLEO, and a (different) extraction of f_{D_s} from BaBar's measurement of $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$ by the Heavy Flavor Averaging Group (HFAG), which have brought the discrepancy down to 2.3σ. Section 3. discusses recent developments in lattice-QCD calculations. New physics explanations are in Sec. 4. The main issues are summarized in Sec. 5.

2. Measurements

Observations of $D_s \to \ell\nu$ date back to 1993 in fixed-target experiments and e^+e^- collisions [12, 13, 14, 15, 16, 17, 18, 19, 20]. These early measurements are omitted from (the current) Particle Data Group (PDG) and HFAG averages and are also omitted from this discussion. In 2006, the PDG [21] included a correlated average of Refs. [16, 17, 18, 19, 20], which increase the discrepancy, as discussed below, by 0.3–0.4σ.

2.1. CLEO $\mu\nu$ and $\tau\nu$

CLEO produces $D_s D_s^{(*)}$ pairs in e^+e^- collisions just above threshold, as in a 1994 observation by the BES Collaboration [14]. The multiplicity is low, so the whole event can be reconstructed, and the neutrino is "detected" by requiring the missing mass-squared to be consistent with 0. Radiative events with photon energy greater than 300 MeV are rejected. Although this cut is imposed for other reasons, it usefully removes radiative events without helicity suppression.

In $D_s \to \tau\nu$, the τ decays in the detector, and the details of the analyses depend on the τ-decay mode. CLEO first observed events in which $\tau \to \pi\nu$ as a background to the $D_s \to \mu\nu$ analysis, but then turned these events into a measurement. A separate analysis chain counts $D_s \to \tau\nu$ events in which $\tau \to e\nu\nu$. With 2 or 3 neutrinos in the final state, the constraint on missing mass-squared is no longer pertinent. These analyses also reject events with photons, but this is a matter of τ detection. In the D_s rest frame, the τ acquires only 9.3 MeV of kinetic energy, so radiative events are not an issue.

In January 2009, CLEO published analyses with their full data-set, reporting

$$f_{D_s} = 257.3 \pm 10.3 \pm 3.9 \text{ MeV} \qquad \mu\nu \qquad [22], \quad (13)$$
$$f_{D_s} = 278.7 \pm 17.1 \pm 3.8 \text{ MeV} \quad \tau\nu, \ \tau \to \pi\nu \ [22], \quad (14)$$
$$f_{D_s} = 252.5 \pm 11.1 \pm 5.2 \text{ MeV} \quad \tau\nu, \ \tau \to e\nu\nu \ [23], \quad (15)$$

which supersede Eq. (7), (9) and (10), respectively. After Physics in Collision 2009, CLEO made public an analysis of a third $D_s \to \tau\nu$ decay chain, $\tau \to \rho\nu$, yielding

$$f_{D_s} = 257.8 \pm 13.3 \pm 5.2 \text{ MeV} \quad \tau\nu, \ \tau \to \rho\nu \ [24]. \quad (16)$$

A novelty of this analysis is that it disentangles a mesa-shaped signal distribution from a peaking background.

2.2. BaBar and Belle $\mu\nu$

BaBar and Belle, following a strategy devised by CLEO [13, 17], collect a D_s sample from continuum events under the $\Upsilon(4S)$ by observing the decay $D_s^* \to D_s\gamma$. BaBar then counts the relative number of $D_s\gamma \to \mu\nu\gamma$ and $D_s\gamma \to \phi\pi\gamma$ events, yielding a measurement of $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$. A separate measurement of $B(D_s \to \phi\pi)$ is needed to extract f_{D_s} via Eq. (3), and BaBar used an average of two of its own measurements [25]. Belle improves on the $D_s^* \to D_s\gamma$ technique by devising a Monte Carlo analysis to guide full reconstruction of the event. In this way they obtain an absolute measurement of $B(D_s \to \mu\nu)$.

Measurements of $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$ are subject to some ambiguity. The ϕ decays to KK, but other processes, such as $D_s \to f_0\pi \to KK\pi$ also contribute. The two contributions are not completely separable, because the amplitudes interfere [26].

2.3. CKM; Radiative Corrections

To extract f_{D_s} from the measurements of the branching ratio, one needs a value of the CKM matrix element $|V_{cd}|$. In practice, it has been determined from CKM unitarity, either using a global fit or simply setting $|V_{cs}| = |V_{ud}|$. (It makes an insignificant difference.) With four or more generations, this assumption incorrect, but 4- (or more) generation CKM unitarity still requires $|V_{cs}| \leq 1$. Therefore, an incorrect assumption about $|V_{cs}|$ cannot explain why the "measured" value of f_{D_s} is too high.

Leptonic decays are, of course, subject to radiative corrections. A class of virtual processes are of special interest here, namely $D_s \to D_s^*\gamma \to \mu\nu\gamma$, where D_s^* is a vector or axial-vector meson. The decay $D_s^* \to \mu\nu$ is not subject to helicity suppression, so the absence in the rate of a factor $(m_\mu/m_{D_s})^2$ could compensate for the presence of the factor $\alpha \approx 1/137$. The radiative rate is significant for energetic photons [27, 28]. With CLEO's cut rejecting radiative events with $E_\gamma > 300$ MeV, however, Eq. (12) of Ref. [27] shows that these events add only around 1% to the rate and, thus, cannot be an explanation of the discrepancy.

2.4. HFAG

The experimental collaborations' differing treatments of $|V_{cs}|$ and of radiative corrections are not yet significant, so Refs. [9, 11] simply average quoted values of f_{D_s}. Eventually, however, a uniform treatment will be necessary, so, with this in mind, the Heavy Flavor Averaging Group (HFAG) [29, 30] has undertaken to average the model-independent quantities $B(D_s \to \mu\nu)$, $B(D_s \to \tau\nu)$, and $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$. The averaging is straightforward. When turning to the extraction of f_{D_s}, however, HFAG noticed an important issue with BaBar's determination of f_{D_s}. The definition of the ϕ resonance in Ref. [5] is a window of K^+K^- invariant mass $M_{K^+K^-}$, such that $|M_{K^+K^-} - m_\phi| < 5.5$ MeV. The normalizing measurements, on the other hand, used $|M_{K^+K^-} - m_\phi| < 15$ MeV [25]. From the $M_{K^+K^-}$ distribution in Ref. [26], it is clear that the difference is important. Fortunately, CLEO [26] reports $B(D_s \to K^+K^-\pi)$ as a function of $M_{K^+K^-}$, so HFAG combines $B(D_s \to K^+K^-\pi)$ with $|M_{K^+K^-} - m_\phi| < 5$ MeV, $B(\phi \to K^+K^-)$, and BaBar's $B(D_s \to \mu\nu)/B(D_s \to \phi\pi)$ to arrive at $B(D_s \to \mu\nu)$. Interpreting this branching fraction as f_{D_s} yields

$$f_{D_s} = 237.3 \pm 16.7 \pm 1.7 \text{ MeV} \ \mu\nu/\phi\pi \text{ HFAG } [30], \quad (17)$$

which we shall use to supersede Eq. (6). It is 16% or 2.9σ lower (using the normalization and systematics for σ).

2.5. Synopsis

In summary, the measurements of the branching fraction $B(D_s \to \ell\nu)$ are relatively straightforward counting experiments. They can be contrasted with, say, searches for the Higgs boson at hadron colliders [31], in which a careful and subtle modeling of the QCD background is essential. Here the background is small and-or measurable; the events are clean, or even pristine. As a result, the dominant experimental error is statistical. It is, of course, possible that more experiments have fluctuated up than down.

With the new results, including Eqs. (16) and (17), the experimental average is now (I find)

$$f_{D_s} = 257.8 \pm 5.9 \text{ MeV}, \quad (18)$$

or 1.7σ lower than Eq. (11), which is a combination of 1.3σ from CLEO's new measurements and 1.1σ from HFAG's revision of BaBar's measurement. (My average without Eq. (16) is 257.8 ± 6.4 MeV, which is close to HFAG's more rigorous average of the same inputs, 256.9 ± 6.8 MeV [30].)

3. Lattice QCD

Lattice QCD has made great strides in the past several years [3, 32], compared to, say, the status at Physics in Collision 2002 [33]. The key development has been the inclusion of sea quarks, first with $n_f = 2$ and, then with $n_f = 2+1$. The latter notation means that one sea quark has a mass nearly equal to that of the strange quark, and the other two vary over a range $0.1 m_s \lesssim m_q \lesssim 0.5 m_s$, such that chiral perturbation theory can be used to reach the up- and down-quark masses.

That said, there have been only two calculations of f_D and f_{D_s} with $n_f = 2+1$ flavors of sea quarks [4, 10], one of which dominates the average. Moreover, both use the same ensembles of lattice gauge fields [32, 34], which have been generated using "rooted staggered fermions" for the sea quarks. The rooting procedure leads to some difficulties [35] that are expected to go away in the continuum limit [36].

The reason for the rooting is that lattice fermion fields correspond to more than one species in the continuum limit [37]. With staggered fermions there are four species [38]. Sea quarks are represented by a determinant of the (lattice) Dirac operator, so to reduce 4 species to 1, one can make the *Ansatz* [39]

$$\left[\det_4(\text{stag} + m)\right]^{1/4} \doteq \det_1(\slashed{D} + m), \qquad (19)$$

where the subscript denotes the number of flavors. The fourth-root can be built into chiral perturbation theory [40]. In fact, in this context the $\frac{1}{4}$ can be replaced by a free parameter, which is then fit. The fit yields 0.28 ± 0.03 [41], in excellent agreement with $\frac{1}{4}$.

The two principle methodological reasons why the error in Eq. (12) is smaller than in Eq. (5) is that Ref. [10] treats the charmed quark as a staggered quark [42], using a pseudoscalar density with an absolute normalization via a PCAC relation [43], and enabling an extrapolation to the continuum limit. By contrast, Ref. [4] treats the charmed quark as a heavy quark [44]; the current requires a matching factor computed in perturbative QCD [45], and the discretization effects are (conservatively) estimated with power-counting estimates [46].

To tackle charm on currently available lattices, the HPQCD Collaboration has developed a highly-improved staggered quark action (HISQ), first used to study charmomium [42]. Some of their other results are tabulated in Table 1. Especially noteworthy here is the value of f_{D^+}, which agrees with CLEO's later measurement [47]. Most effects that would bring Eq. (12) into better agreement with the measurements of f_{D_s} would also alter f_{D^+}, spoiling its agreement.

For the π, K, and D^+ decay constants, both the chiral and continuum extrapolations are crucial. For the D_s, however, the valence charmed and strange quarks ensure a mild chiral extrapolation. The continuum extrapolation turns out to be interesting: reading values for f_{D_s} off of plots in Ref. [10], I have verified the continuum extrapolation and found that the slope in a^2 conforms with expectations of discretization effects of order $\alpha_s a^2 m_c \Lambda$.

The relevant portion of HPQCD's error budget is presented in Table 2. Most of the row headings are self-explanatory, except for "scale r_1," which is discussed below. The error budget is nearly complete, in my opinion,

Table 1. Results from Refs. [10, 42] other than f_{D_s}. Experimental quantities are taken from Ref. [11], except for f_{D^+}, which is from Ref. [47]. Here $\Delta_q = 2m_{D_q} - m_{\eta_c}$, $q = d, s$.

Quantity	Expt	HPQCD	units
$m_{J/\psi} - m_{\eta_c}$	116.4 ± 1.2	111 ± 5	MeV
Δ_d	758.7	755 ± 14	MeV
Δ_s	956.5	944 ± 12	MeV
Δ_s/Δ_d	1.261 ± 0.002	1.252 ± 0.015	
f_π	130.7 ± 0.4	132 ± 2	MeV
f_K	159.8 ± 0.5	157 ± 2	MeV
f_{D^+}	205.8 ± 8.9	207 ± 4	MeV

Table 2. Error budget from Ref. [10]. Entries in percent.

Source	f_{D_s}	f_D	f_{D_s}/f_D
Statistics	0.6	0.7	0.5
Scale r_1	1.0	1.4	0.4
Continuum limit	0.5	0.6	0.4
Chiral limit	0.3	0.4	0.2
Adjust m_s	0.3	0.3	0.3
Adjust $m_d \oplus$ QED	0.0	0.1	0.1
Finite volume	0.1	0.3	0.3

more complete than many error budgets in the lattice-QCD literature. It does, however, fail to quote an uncertainty for quenching the charmed sea. This is surely a small effect, of order $\alpha_s(\Lambda/m_c)^2$, but perhaps commensurate with the $\frac{1}{2}\%$ errors included in Table 2.

I shall now discuss r_1 in several steps, first motivating why it is used, then giving its definition and its value circa 2007. Being based on an expanding set of numerical data, its value has now changed, so I discuss how it affects charmed-meson decay constants.

Lattice gauge theory has a built-in ultraviolet cutoff—the lattice itself. The natural output is a dimensionless number, with physical dimensions balanced by powers of the lattice spacing a. With a decay constant f, one computes af and then must introduce a definition for a. This is necessary not merely to quote a final result in MeV, but also to combine calculations at varying a, which is needed to understand the continuum limit. This is done by picking some fiducial mass M, and defining $a = (aM)_{\text{lat}}/M_{\text{expt}}$. This step eliminates one of the free parameters of QCD, namely, the bare coupling.

To keep a long story short, no quantity is ideally suited to play the role of M. A popular choice is $1/r_1$, defined via [48, 49]

$$r_1^2 F(r_1) = 1, \qquad (20)$$

where $F(r)$ is the force between two static sources of color, distance r apart. The advantages of r_1 are that it is easy to compute in lattice QCD, and that it depends weakly on sea-quark masses and not at all on valence-quark masses. Then one can combine data from several lattices for $r_1 f = (r_1/a)(af)$ in the chiral and continuum extrapolations. Other choices of M could complicate these steps.

Of course, r_1 is unknown—it cannot be measured in the lab. It is inferred from the chiral and continuum

limit of other quantities. Reference [10] used the value

$$r_1 = 0.321 \pm 0.005 \text{ fm}, \qquad (21)$$

based on MILC's calculations of r_1/a [34] and HPQCD's own calculations of $a(M_{\Upsilon(2S)} - M_{\Upsilon(1S)})$ [50]. The 1.6% uncertainty in r_1 translates into a 1.0% uncertainty on f_{D_s} (c.f. Table 2.), because when r_1 varies, the bare valence quark masses inside the D_s do too.

This retuning when r_1 changes has been studied by the Fermilab Lattice and MILC Collaborations (although not all details are as yet public). Since Ref. [4] was published, MILC has extended the ensembles to higher statistics, so Fermilab/MILC's decay constant analysis has continued, to reduce the total error. At Lattice 2008 some of the discretization errors were brought under better control, leading to 249 ± 11 MeV [51], with (serendipitously) the same central value as Eq. (5). References [4, 51] used $r_1 = 0.318 \pm 0.007$ fm [52] based on essentially the same input information as Eq. (21).

Meanwhile, however, evidence has begun to accumulate that r_1 should be smaller. Focusing on MILC's latest analysis of $r_1 f_\pi$ [53], one has

$$r_1 = 0.3108 \pm 0.0022 \text{ fm}. \qquad (22)$$

Retuning the quark masses, this changes f_{D_s} to (preliminary, presented at Lattice 2009)

$$f_{D_s} = 260 \pm 10 \text{ MeV Fermilab/MILC [52]}, \qquad (23)$$

in which 4.2 MeV of the increase stems from the change in r_1, and the rest from other refinements of the analysis [52]. In other words, a shift down of 2.3% in r_1 has led to a shift up of 1.7% in f_{D_s}.

The HPQCD Collaboration has also incorporated the extensions of the MILC ensembles into its analysis of r_1 [54]. They find

$$r_1 = 0.3133 \pm 0.0023 \text{ fm}, \qquad (24)$$

which is 2.4% lower than the value in Eq. (21). Although this suggests an increase in f_{D_s} of 3–5 MeV, one should keep in mind that HPQCD's calculations of f_{D_s} have proceeded to yet finer lattices. It seems prudent to wait for their own update, rather than applying a shift.

Because both Fermilab/MILC and HPQCD use the same ensembles of lattice gauge fields, it is, or should be, a high priority to compute f_D and f_{D_s} with other formulations of sea quarks. A promising development comes from the European Twisted-Mass Collaboration (ETMC), which has ensembles with $n_f = 2$ over a range of sea-quark masses and lattice spacings (although not as extensive as MILC's with $n_f = 2 + 1$). They find $f_{D_s} = 244 \pm 8$ MeV [55], where the error stems from a thorough analysis of all uncertainties *except* the quenching of the strange quark. It is not easy to estimate this error reliably enough for averaging. Earlier results with $n_f = 2$ obtained similar central values [56], or a bit higher [57], albeit with larger error bars.

It seems reasonable, then, to take as the current best estimate from lattice QCD, the weighted average of Eqs. (12) and (23):

$$f_{D_s} = 242.6 \pm 2.9 \text{ MeV} \quad \text{LQCD 2009}, \qquad (25)$$

which is 2.3σ lower than the average of measurements in Eq. (18). The experimental statistical error continues to dominate this σ, although if the central value of the lattice average were to increase by 3–5 MeV, the discrepancy would soften below 2σ.

4. New Physics

The foregoing discussion makes clear that it is desirable both for the experiments to improve further in precision and for the lattice-QCD calculations to be confirmed. Given the current status, it is conceivable that the tension will increase again to the point that it warrants broad attention. With that in mind, this section provides some information on extensions of the Standard Model.

The decays $D_s \to \ell\nu$ could be mediated by particles other than the Standard W, either by s-channel annihilation via another charge-$+1$ particle, by t-channel exchange of a charge-$+\frac{2}{3}$ particle, or by u-channel exchange of a charge-$-\frac{1}{3}$ particle. All three kinds of particle are popular enough in extensions of the Standard Model to have their own sections in the Review of Particle Physics [11, 21]. The charge-$+1$ particle would be a W' or a charged Higgs boson; the fractionally charged particles are known as leptoquarks. All would have a mass, presumably, at least as large as M_W. Their interactions can be parametrized by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \sqrt{2} G_A^\ell (\bar{s}\gamma^\mu\gamma_5 c)(\bar{\nu}_L\gamma_\mu\ell_L) + \sqrt{2} G_P^\ell (\bar{s}\gamma_5 c)(\bar{\nu}_L\ell_R)$$
$$- \sqrt{2} G_V^\ell (\bar{s}\gamma^\mu c)(\bar{\nu}_L\gamma_\mu\ell_L) + \sqrt{2} G_S^\ell (\bar{s}c)(\bar{\nu}_L\ell_R)$$
$$+ \sqrt{2} G_T^\ell (\bar{s}\sigma^{\mu\nu}c)(\bar{\nu}_L\sigma_{\mu\nu}\ell_R), \qquad (26)$$

where G_A^l and G_P^l appear in the leptonic-decay amplitude (4). The other interactions are likely to arise in non-Standard models, stemming from the chiral quantum numbers of the quarks and leptons. Nonzero G_V^ℓ would interfere with the leading Standard amplitude of the semileptonic decay $D \to K\ell\nu$, potentially making a significant change in the rate [58]. On the other hand, nonzero G_S^ℓ or G_T^ℓ would interfere with helicity-suppression, being visible only in an asymmetry of $D \to K\mu\nu$ after 10^7 or more events are recorded [58].

A W' alters the (semi)leptonic amplitude via G_A (G_V). Barring a carefully-built (*i.e.*, finely-tuned) model, this is not a promising scenario [9]. Many popular charged Higgs models are also unpromising. Reference [9] presents a charged Higgs model that could explain an excess of $D_s \to \ell\nu$ events, but it predicts the same-sized excess in $D^+ \to \ell\nu$. Now, however, this is disfavored by the near-perfect agreement the most precise measurement of f_{D^+} [47] with lattice QCD [4, 10, 52, 55].

Leptoquarks, of several ilks, remain. Even here the charge-$+\frac{2}{3}$ case is unpromising [9], owing to constraints from the lepton-flavor violating decays $\tau \to \mu\bar{s}s$, where $\bar{s}s$ hadronizes to ϕ or KK. This leaves the most promising candidate to be an SU(2)-singlet, charged-$-\frac{1}{3}$ leptoquark. This particle has the quantum numbers of a scalar down quark $\tilde{d}$, with an R-violating interaction

$$(\kappa_{c\ell}\bar{c}_L\ell_L^c - \kappa_{q\ell}V_{qs}^*\bar{s}_L\nu_{\ell L}^c)\tilde{d} + \kappa_{c\ell}'\,\bar{c}_R\ell_R^c\tilde{d} + \text{H.c.}, \qquad (27)$$

where the superscript "c" denotes charge conjugation, and κ and κ' are coupling matrices. (With down squarks, the $\tilde{d}$ field should take a family index, and κ and κ' yet another index.) Exchange of $\tilde{d}$ generates Eq. (26) with

$$G_A^\ell = G_V^\ell = \kappa_{c\ell}^*\kappa_{ql}V_{qs}^*/4\sqrt{2}M_{\tilde{d}}^2, \qquad (28)$$

$$G_P^\ell = G_S^\ell = \kappa_{c\ell}'^*\kappa_{ql}V_{qs}^*/4\sqrt{2}M_{\tilde{d}}^2 = 2G_T^\ell. \qquad (29)$$

Generalizations of Eq. (27) appear in non-Standard models that arise in many contexts [59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74].
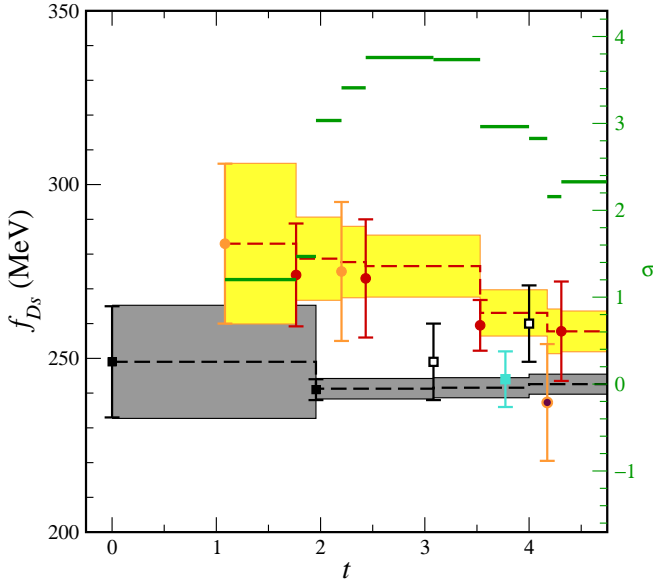
Fig. 1. Historical development of the f_{D_s} puzzle since June 2005, in years since Ref. [4] was posted at arXiv.org. Circles denote experimental measurements, from e^+e^- collisions at the $\Upsilon(4S)$ in medium grey (orange); from e^+e^- collisions at the $D_s^{(*)}D_s^{(*)}$ threshold in dark grey (red); their weighted average is the medium grey (orange) dashed line with error band in light grey (yellow). Squares denote lattice-QCD calculations, with $n_f = 2$ flavors in medium grey (cyan); with $n_f = 2 + 1$ flavors in black (open symbols for conference reports); the $2 + 1$ weighted average is the black dashed line with error band in grey. The discrepancy σ is shown by the grey (green) solid line segments, labeled by the vertical axis on the right. (color online)

The leptoquark possibility been examined in a broader context including constraints from decays of the D meson, kaon, τ lepton, and proton [75]. Reference [75] claims these constraints make it difficult for the interaction (27) to explain an enhancement in both $B(D_s \to \mu\nu)$ and $B(D_s \to \tau\nu)$ at the same time.

5. Summary

The developments of the f_{D_s} puzzle are collected into Fig. 1., which presents the measurements and calculations discussed above, and the time dependence of their respective averages. The time axis starts with the posting of Ref. [4], the first lattice-QCD calculation with $2 + 1$ flavors of sea quarks. The CLEO results of Ref. [6] ($t = 1.8$) and of Refs. [22, 23] ($t = 3.5$) each are averaged to reduce clutter.

The discrepancy arose mostly with HPQCD's lattice-QCD calculation, and partly with the 2007 measurements from Belle and CLEO, rising to 3.8σ. In the past 18 months, the tension in f_{D_s} has fallen to 2.3σ, from

- CLEO's new measurements of January 2009: -0.8σ;

- Fermilab/MILC's (preliminary) update for Lattice 2009: -0.13σ;

- HFAG's reinterpretation of BaBar's measurement: -0.67σ.

- CLEO's new measurement of October 2009: $+0.1\sigma$;

The history of the discrepancy's significance is traced via the grey (green) piece-wise horizontal line and right vertical axis in Fig. 1. Now, by the way, if one follows Ref. [11] and omits the BaBar result (as reinterpreted by HFAG), the discrepancy is 2.6σ.

The prospects for a resolution of the f_{D_s} puzzle are good—whether the tension goes away completely or tightens again. BaBar is measuring the absolute branching ratio, and Belle plans to update its analysis with higher statistics. In a few years, BES 3 will measure $D_s \to \ell\nu$ in threshold production, similarly to CLEO, with a target uncertainty of 1% [76]. Several lattice-QCD collaborations now have enough $n_f = 2+1$ ensembles to carry out a useful calculation of f_{D_s}. The MILC Collaboration has started to generate ensembles with $n_f = 2 + 1 + 1$ sea quarks with the HISQ action, where the fourth sea quark is charm. Similarly, the ETMC has embarked on a project with $n_f = 2 + 1 + 1$ twisted-mass Wilson sea quarks. Even if the puzzle dissipates, D and D_s leptonic decays will be useful for constraining extensions of the Standard Model [75].

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